THICK SHELL TRIANGULAR FINITE ELEMENT FOR GEOMETRICALLY NONLINEAR ANALYSIS

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This paper presents the formulation of thick shell finite elements for geometrically nonlinear analysis. The derivation of the geometric stiffness matrix that is of paramount importance in any nonlinear analysis is performed by load perturbing the discretized equilibrium equations of an element in its local coordinate system, following Levy and Spillers [1], to yield the inplane geometric stiffness matrix. Out of plane considerations that involve rigid body rotations will finally provide an additional geometric stiffness matrix to be included when assembling the tangential stiffness matrix. Rotations are considered as finite according to Rodrigues, and rigid body motion is elegantly removed using a unique procedure to result in pure elastic deformations that enable stress retrieval via linear constitutive relations. The discrete Kirchhoff theory (DKT) triangular plate element of Batoz *et al* [2] which was extended by Katili [3] to include plate bending based on Mindlin-Reissner plate theory and the constant stress triangular membrane element are chosen to fulfill the bending and membrane needs of the shell respectively. This derivation is complete in the sense that all contributions to the response of the same order of magnitude are included.

Three ways may be found in the literature for deriving the geometric stiffness matrix in finite element shell analysis: 1) that of Crisfield [4], among others, who proceeded with a co-rotational manner for nonlinear analysis, 2) the 3-D elasticity "degenerate" element with the nonlinear contributions of, among others, Hughes and Lui [5] and 3) the interesting work of Yang and Chang [6] who derive the geometrical stiffness matrix from the rigid body rotations only. This paper constitutes the fourth.

Whereas the presented approach depends on an a priori chosen linear elastic finite element it is independent of large strain formulations that are essential otherwise. Finally a rich ensemble of well-documented benchmark examples is provided to demonstrate the validity and success of the proposed formulation.

References

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